

	Number Line	Strip	Pie	Grid
General	Conceptual, No Final Values	Procedural, Math Shown	Procedural, Math Shown	Hybrid, Some Math Visual
Compare > <	Lengths	Area	Area	Area, $\frac{3}{4}$ and $\frac{2}{3}$ create a visually interesting case
Addition +	0 of second line aligned with endpoint of first line	Common Denominators created by using other denominator. Identical denominators remain the same.	Same as strip except numbers greater than one have whole parts move as wholes first, then fractional parts move.	Creates common denominators through visually cross hatch. Identical denominators remain the same.
Subtraction -	Second line endpoint aligns with first line endpoint.	Parts from second overlap and remove parts from the first.	Wholes cancel first, fractional parts cancel second.	Parts from second overlap and remove parts from the first.
Multiplication x	Treats the first fraction as a unit and then takes second fraction of that unit. Multiplication is viewed as scaling.	Creates fractional parts based on the multiplier denominator and then accumulates multiplier numerator of these parts. See longer explanation*.	Same as Strip.	Treats one fraction as the base and then the second as a cross-section of that base. Suggested entry progression: $\frac{1}{2} \times 3$, $\frac{1}{2} \times 2$, $\frac{1}{2} \times 1$, $\frac{1}{2} \times \frac{1}{2}$
Benchmark ← →	Length comparison to benchmark number (default $\frac{1}{2}$)	Area comparison to benchmark number.	Same as Strip.	Magically creates least common denominator of two fractions and benchmark before comparison.

- Explanation of Multiplication representation for Strip and Pie.

For the case of $\frac{1}{4} \times \frac{2}{3}$ there are two natural choices:

- 1) Take 2 fourths and then partition by three. One of these partitions is your answer. $2(\frac{1}{4}) / 3$
- 2) Partition $\frac{1}{4}$ by three and then take two of these partitions. $(\frac{1}{4} / 3) * 2$

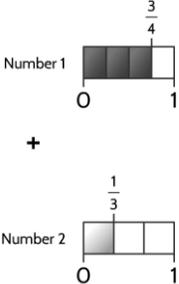
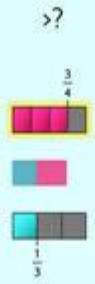
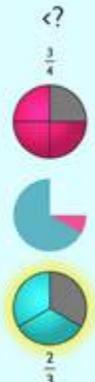
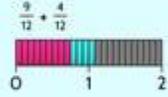
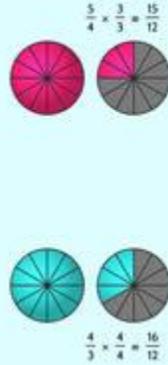
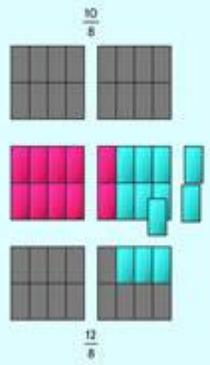
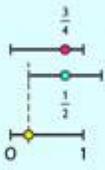
The first option above becomes more complicated if $\frac{1}{4}$ is replaced with a non-unit fraction such as $\frac{3}{4}$. If you have the intermediary value of $2(\frac{3}{4})$ and then partition it into 3 parts it may be clear what portion is being taken but it's not clear why the denominator is now 12ths.

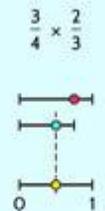
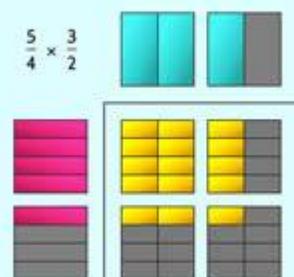
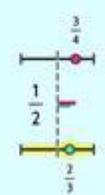
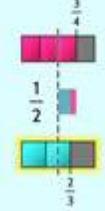
The second option is not much better (although in this particular example it's easy to break $\frac{3}{4}$ into 3 parts).

Ultimately it was decided to adopt the second option but in a distributive manner.

Instead of doing $(\frac{3}{4} / 3) * 2$, we decide to do $(\frac{1}{4} + \frac{1}{4} + \frac{1}{4}) / 3 * 2$ taking a third of each of the fourths and then taking 2 parts from each.

That this method requires so much explanation indicates that it is probably NOT the best way to show multiplication. Number line or grid may prove more useful in the classroom. Suggestions of how to better show multiplication with pie or strip with both proper and improper fractions are greatly welcome.

	Number Line	Strip	Pie	Grid
General	Conceptual, No Final Values	Procedural, Math Shown	Procedural, Math Shown	Hybrid, Some Math Visual
Compare > <		<p>Greater: $\frac{3}{4} > \frac{1}{3}$ Click here to toggle the menu</p> 	<p>Lesser: $\frac{3}{4} > \frac{1}{3}$ Click here to toggle the menu</p> 	
Addition +		<p>Addition: $\frac{9}{12} + \frac{4}{12}$ Click here to toggle the menu</p> 	<p>Addition: $\frac{5}{4} + \frac{3}{3} = \frac{15}{12}$ Click here to toggle the menu</p> 	<p>Addition: $\frac{10}{8} + \frac{12}{8}$ Click here to toggle the menu</p> 
Subtraction -	<p>Subtraction: $\frac{3}{4} - \frac{1}{3}$ Click here to toggle the menu</p> 			

<p>Multiplication x</p>	<p>Multiplication: $\frac{3}{4} \times \frac{2}{3}$ Click here to toggle the menu</p>  <p>A number line from 0 to 1 with tick marks at 0, 1/4, 2/4, 3/4, and 1. A red dot is at 3/4. A dashed vertical line goes down from 3/4 to a yellow dot at 2/3 on the number line. A horizontal line segment is drawn from 0 to 2/3, divided into three equal parts by a vertical dashed line.</p>	<p>Multiplication: $\frac{3}{4} \times \frac{3}{2}$ Click here to toggle the menu</p>  <p>A horizontal bar divided into four equal parts, with the first three parts shaded pink. Below it, a vertical bar divided into two equal parts, with the left part shaded yellow. A small yellow rectangle is placed at the bottom right corner of the pink area.</p>		<p>Multiplication: $\frac{5}{4} \times \frac{3}{2}$ Click here to toggle the menu</p>  <p>A grid of 20 squares arranged in 4 rows and 5 columns. The top two rows are shaded pink. The bottom two rows are shaded yellow. A vertical dashed line is drawn between the second and third columns. A horizontal dashed line is drawn between the second and third rows. A small yellow rectangle is placed at the bottom right corner of the pink area.</p>
<p>Benchmark ← →</p>	<p>Benchmark: $\frac{2}{3}$ is closer to $\frac{1}{2}$ Click here to toggle the menu</p>  <p>A number line from 0 to 1 with tick marks at 0, 1/2, 2/3, 3/4, and 1. A red dot is at 3/4. A yellow dot is at 2/3. A dashed vertical line goes down from 3/4 to the yellow dot at 2/3. A horizontal line segment is drawn from 0 to 2/3, divided into three equal parts by a vertical dashed line.</p>	<p>Benchmark: $\frac{2}{3}$ is closer to $\frac{1}{2}$ Click here to toggle the menu</p>  <p>A horizontal bar divided into four equal parts, with the first three parts shaded pink. Below it, a vertical bar divided into two equal parts, with the left part shaded yellow. A small yellow rectangle is placed at the bottom right corner of the pink area.</p>		

- Explanation of Multiplication representation for Strip and Pie.

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Benchmark ← →				

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Benchmark: $\frac{1}{2} < 1$ Click here to toggle the menu

$\frac{8}{12}$ is closer to $\frac{1}{2}$

$\frac{8}{12}$

Fractionator ALP14

Greater: $>$ Click here to toggle the menu

$\frac{3}{4} > \frac{2}{3}$

$\frac{3}{4}$

$\frac{2}{3}$

Fractionator ALP14

Subtraction: $-$ Click here to toggle the menu

$\frac{8}{12}$

$\frac{2}{12}$

$\frac{6}{12}$

Addition: $+$ Click here to toggle the menu

$\frac{3}{4} + \frac{3}{4} = \frac{6}{4}$

$\frac{3}{4}$

$\frac{3}{4}$

$\frac{6}{4}$

$\frac{2}{3} + \frac{4}{4} = \frac{8}{12}$

$\frac{2}{3}$

$\frac{4}{4}$

$\frac{8}{12}$

Fractionator ALP14

Multiplication: \times Click here to toggle the menu

$\frac{3}{4} \times \frac{2}{3}$

$\frac{3}{4}$

$\frac{2}{3}$

$\frac{4}{6}$

Fractionator ALP14

Addition: $+$ Click here to toggle the menu

$\frac{10}{8}$

$\frac{10}{8}$

$\frac{10}{8}$

Multiplication: \times Click here to toggle the menu

$\frac{5}{4} \times \frac{2}{3}$

$\frac{5}{4}$

$\frac{2}{3}$

$\frac{10}{6}$

Fractionator ALP14

Addition: $+$ Click here to toggle the menu

Number 1: $\frac{1}{4}$

Number 2: $\frac{3}{4}$

$\frac{1}{4}$

$\frac{3}{4}$

$\frac{4}{4}$

Fractionator ALP14

Addition: $+$ Click here to toggle the menu

$\frac{1}{4}$

$\frac{3}{4}$

1

Benchmark: $\frac{1}{2} < 1$ Click here to toggle the menu

$\frac{2}{3}$ is closer to $\frac{1}{2}$

$\frac{2}{3}$

$\frac{1}{2}$

1

Greater: $>$ Click here to toggle the menu

$\frac{3}{4} > \frac{2}{3}$

$\frac{3}{4}$

$\frac{2}{3}$

Subtraction: $-$ Click here to toggle the menu

$\frac{3}{4}$

$\frac{1}{4}$

$\frac{2}{4}$

$\frac{1}{2}$

Subtraction: $\frac{3}{4} - \frac{1}{4}$

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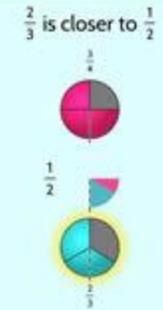
Multiplication: $\frac{3}{4} \times \frac{2}{3}$

Click here to toggle the menu



Benchmark: $\frac{2}{3} > \frac{1}{2}$

Click here to toggle the menu



Lesson: $\frac{3}{4} > \frac{1}{2}$

Click here to toggle the menu



Subtraction: $\frac{6}{12} - \frac{1}{12}$

Click here to toggle the menu



Addition: $\frac{3}{4} + \frac{1}{4}$

Click here to toggle the menu



Multiplication: $\frac{3}{4} \times \frac{2}{3}$

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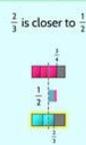
Addition: $\frac{1}{3} + \frac{2}{6}$

Click here to toggle the menu



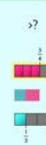
Benchmark: $\frac{2}{3} > \frac{1}{2}$

Click here to toggle the menu



Greater: $\frac{1}{2}$

Click here to toggle the menu



Subtraction: $\frac{9}{12} - \frac{1}{12}$

Click here to toggle the menu



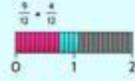
Addition $\frac{1}{4} + \frac{1}{4}$

[Click here to toggle the menu](#)



Addition $\frac{3}{10} + \frac{4}{10}$

[Click here to toggle the menu](#)



Multiplication $\frac{3}{4} \times \frac{2}{3}$

[Click here to toggle the menu](#)

$$\frac{3}{4} \times \frac{2}{3}$$

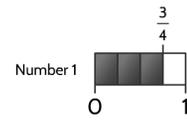
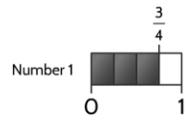
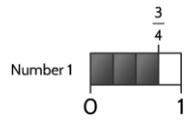
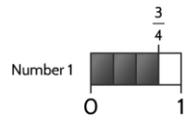


Multiplication $\frac{3}{4} \times \frac{3}{2}$

[Click here to toggle the menu](#)

$$\frac{3}{4} \times \frac{3}{2}$$



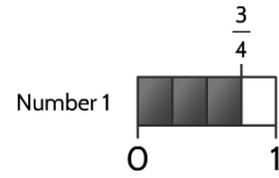
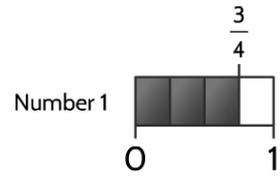
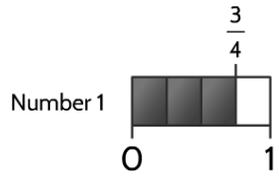
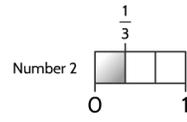
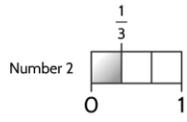
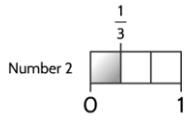
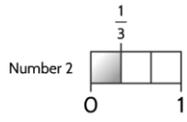


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